

The pure annihilation type decays $B^0 \rightarrow D_s^- K_2^{*+}$ and $B_s \rightarrow \bar{D}a_2$ in perturbative QCD approach

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Abstract

We calculate the branching ratios of pure annihilation type decays $B^0 \rightarrow D_s^- K_2^{*+}$ and $B_s \rightarrow \bar{D}a_2$ using the perturbative QCD approach based on k_T factorization. The branching ratios are predicted to be $(60.6_{-16.5}^{+17.3} {}_{-10.4}^{+4.3} {}_{-2.1}^{+3.2}) \times 10^{-6}$ for $B^0 \rightarrow D_s^- K_2^{*+}$, $(1.1_{-0.4}^{+0.4} {}_{-0.2}^{+0.1} {}_{-0.1}^{+0.1}) \times 10^{-6}$ for $B_s \rightarrow \bar{D}a_2^0$ and $(2.3_{-0.8}^{+0.8} {}_{-0.4}^{+0.2} {}_{-0.1}^{+0.1}) \times 10^{-6}$ for $B_s \rightarrow D^- a_2^+$. They are large enough to be measured in the ongoing experiment. Due to the shortage of contributions from penguin operators, there are no direct CP asymmetries for these decays in the standard model. We also derive simple relations among these decay channels to reduce theoretical uncertainties for the experiments to test the accuracy of theory and search of new physics signal.

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I. INTRODUCTION

Two body hadronic B decays have been a hot topic for many years, since it involves the perturbative QCD calculation and factorization study. It is also important for the test of standard model, the CKM angle measurements and the search of new physics phenomena. For many years, people do calculations based on the naive factorization assumption, later proved by the soft-collinear effective theory[1]. However, there is one kind of diagrams, the so called annihilation type diagrams, which was argued to be helicity suppressed since no one knows how to calculate. In the well developed collinear factorization, there is endpoint singularity in the calculation of these diagrams. In fact, this kind of diagrams are essential for the strong phase of direct CP asymmetry in the $B \rightarrow K^+\pi^-$ decays [2], which is proved to be important.

Furthermore, there is one kind of B decays, which contains only annihilation type diagram contributions. One of the examples is the $B^0 \rightarrow D_s K^+$ decay, which is predicted in ref.[3–5] and measured by the B factories later [6]. Recently, the CDF collaboration measured the first pure annihilation type decays in the B_s sector i.e. $B_s \rightarrow \pi^+\pi^-$ decay, which exactly confirms the perturbative QCD prediction for this decay [7, 8]. It is worth of mentioning that the perturbative QCD (PQCD) approach is almost the only method can do the quantitative calculations of the annihilation type diagrams [7, 8].

In this paper, we shall study the pure annihilation type charmed decays $B^0 \rightarrow D_s^- K_2^{*+}$ and $B_s \rightarrow \bar{D}a_2$ in the PQCD approach, which is based on the k_T factorization [9, 10]. These decays are predicted to have a large branching ratio as 10^{-6} to 10^{-5} , which are measurable in the near future experiments. In the annihilation type diagrams, both of the light quark and the heavy anti b quark in B meson annihilate into another quark anti-quark pair through the four quark operators, while another light quark pair in the final state mesons are produce by a gluon attaching to the four quark operator. Since the light quark in the final states are collinear, the gluon connection them must be hard. So the hard part of the PQCD approach contains six quarks rather than four quarks. This is called six-quark effective theory or six-quark operator. In this approach, the quarks' intrinsic transverse momenta are kept to avoid the endpoint divergence. Because of the additional energy scale introduced by the transverse momentum, double logarithms will appear in the QCD radiative corrections. We resum these double logarithms to give

a Sudakov factor, which effectively suppresses the end-point region contribution. This makes the PQCD approach more reliable and consistent.

This paper is organized as following. In Sec.II, we present the formalism and perform the perturbative calculations for considered decay channels with the PQCD approach. The numerical results and phenomenological analysis are given in Sec.III. Finally, Sec.IV contains a short summary.

II. FORMALISM AND PERTURBATIVE CALCULATION

The $B^0 \rightarrow D_s^- K_2^{*+}$, $B_s \rightarrow \bar{D}^0 a_2^0$ and $B_s \rightarrow D^- a_2^+$ decays are pure annihilation type rare decays. At the quark level, these decays are described by the effective Hamiltonian H_{eff} [11]

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uD} [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)], \quad (1)$$

where V_{cb} and V_{uD} are CKM matrix elements, “ D ” denotes the light down quark d or s , and $C_{1,2}(\mu)$ are Wilson coefficients at the renormalization scale μ . $O_{1,2}(\mu)$ are the four quark operators.

$$O_1 = (\bar{b}_\alpha c_\beta)_{V-A} (\bar{u}_\beta D_\alpha)_{V-A}, \quad O_2 = (\bar{b}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta D_\beta)_{V-A}. \quad (2)$$

where α and β are the color indices, $(\bar{b}_\alpha c_\beta)_{V-A} = \bar{b}_\alpha \gamma^\mu (1 - \gamma^5) c_\beta$. Conventionally, we define the combined Wilson coefficients as

$$a_1 = C_2 + C_1/3, \quad a_2 = C_1 + C_2/3. \quad (3)$$

In the hadronic matrix element calculation, we factorize the decay amplitude into soft(Φ), hard(H), and harder (C) dynamics characterized by different scales [12, 13],

$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \times Tr [C(t) \Phi_B(x_1, b_1) \Phi_{M_2}(x_2, b_2) \Phi_{M_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)}]. \quad (4)$$

where b_i is the conjugate variable of quark’s transverse momentum k_{iT} , x_i is the momentum fractions of valence quarks, and t is the largest energy scale in function $H(x_i, b_i, t)$ which is the hard part. $C(t)$ are the Wilson coefficients with resummation of the large logarithms $\ln(m_W/t)$ produced by the QCD corrections of four quark operators. $S_t(x_i)$

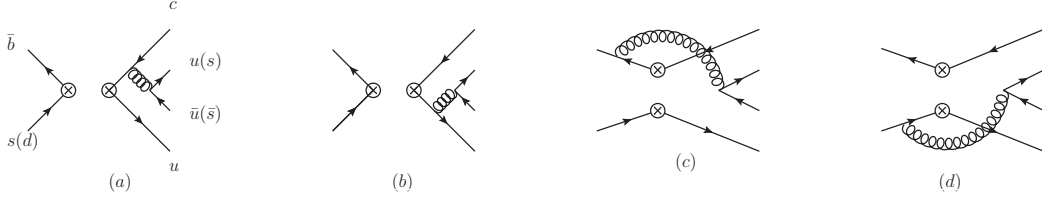


FIG. 1: annihilation diagrams contributing to the $B \rightarrow \bar{D}T$ decays in PQCD

is the jet function, which is obtained by the threshold resummation and smears the end-point singularities on x_i [14]. The last term, $e^{-S(t)}$, is the Sudakov form factor, from resummation of double logarithms, which suppresses the soft dynamics effectively and the long distance contributions in the large b region [15, 16]. Thus it makes the perturbative calculation of the hard part H applicable at intermediate scale, i.e., m_B scale. The Φ_i , meson wave functions, are nonperturbative input parameters but universal for all decay modes.

The lowest order Feynman diagrams of the considered decays are shown in Fig.1. The amplitude from factorizable diagrams (a) and (b) in Fig.1 is

$$\begin{aligned}
\mathcal{A}_{af} = & 8\sqrt{\frac{2}{3}}C_F f_B \pi m_B^4 \int_0^1 dx_2 dx_3 \int_0^{1/\Lambda} b_2 db_2 b_3 db_3 \phi_D(x_2, b_2) \\
& \times \{ [-\phi_T(x_3)x_2 + 2r_D r_T \phi_T^s(x_3)(x_2 + 1)] \\
& \cdot h_{af}((1 - x_3), x_2(1 - r_D^2), b_2, b_3) E_{af}(t_e) \\
& - [\phi_T(x_3)(x_3 - 1) - r_D r_T (\phi_T^t(x_3)(1 - 2x_3) + \phi_T^s(x_3)(2x_3 - 3))] \\
& \cdot h_{af}(x_2, (1 - x_3)(1 - r_D^2), b_3, b_2) E_{af}(t_f) \}.
\end{aligned} \tag{5}$$

In this function, $C_F = 4/3$ is the group factor of $SU(3)_c$, and $r_{D(T)} = m_{D(T)}/m_B$. The hard scale $t_{e,f}$ and the functions E_{af} and h_{af} are given by

$$\begin{aligned}
t_e &= \max\{\sqrt{x_2(1 - r_D^2)m_B}, 1/b_2, 1/b_3\}, \\
t_f &= \max\{\sqrt{(1 - x_3)(1 - r_D^2)m_B}, 1/b_2, 1/b_3\},
\end{aligned} \tag{6}$$

$$E_{af}(t) = \alpha_s(t) \cdot \exp[-S_T(t) - S_D(t)], \tag{7}$$

$$\begin{aligned}
h_{af}(x_2, x_3, b_2, b_3) = & \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_2 x_3} m_B b_2) \\
& \left[\theta(b_2 - b_3) H_0^{(1)}(\sqrt{x_3} m_B b_2) J_0(\sqrt{x_3} m_B b_3) + \right. \\
& \left. \theta(b_3 - b_2) H_0^{(1)}(\sqrt{x_3} m_B b_3) J_0(\sqrt{x_3} m_B b_2) \right] \cdot S_t(x_3). \quad (8)
\end{aligned}$$

The amplitude for nonfactorizable diagrams (c) and (d) in Fig.1 is

$$\begin{aligned}
\mathcal{M}_{anf} = & \frac{32}{3} C_F \pi m_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_D(x_2, b_2) \\
& \times \left\{ [\phi_T(x_3) x_2 + r_D r_T (\phi_T^s(x_3)(x_3 - x_2 - 3) + \phi_T^t(x_3)(x_2 + x_3 - 1))] \right. \\
& \cdot h_{anf1}(x_1, x_2, x_3, b_1, b_2) E_{anf}(t_g) \\
& + [\phi_T(x_3)(x_3 - 1) + r_D r_T (\phi_T^s(x_3)(x_2 - x_3 + 1) + \phi_T^t(x_3)(x_2 + x_3 - 1))] \\
& \cdot h_{anf2}(x_1, x_2, x_3, b_1, b_2) E_{anf}(t_h) \left. \right\}. \quad (9)
\end{aligned}$$

$$\begin{aligned}
t_g = & \max\{\sqrt{x_2(1-x_3)(1-r_D^2)} m_B, \sqrt{1-(1-(1-x_3)(1-r_D^2))(1-x_1-x_2)} m_B, \\
& 1/b_1, 1/b_2\}, \\
t_h = & \max\{\sqrt{x_2(1-x_3)(1-r_D^2)} m_B, \sqrt{(1-x_3)(1-r_D^2)} |x_1 - x_2| m_B, \\
& 1/b_1, 1/b_2\}, \quad (10)
\end{aligned}$$

$$E_{anf} = \alpha_s(t) \cdot \exp[-S_B(t) - S_T(t) - S_D(t)] \mid_{b_2=b_3}, \quad (11)$$

$$\begin{aligned}
h_{anfj}(x_1, x_2, x_3, b_1, b_2) = & \frac{i\pi}{2} \left[\theta(b_1 - b_2) H_0^{(1)}(F m_B b_1) J_0(F m_B b_2) \right. \\
& + \theta(b_2 - b_1) H_0^{(1)}(F m_B b_2) J_0(F m_B b_1) \left. \right] \\
& \times \begin{cases} \frac{i\pi}{2} H_0^{(1)}(\sqrt{|F_j^2|} m_B b_1), & F_j^2 < 0, \\ K_0(F_j m_B b_1), & F_j^2 > 0, \end{cases} \quad (12)
\end{aligned}$$

with $j = 1, 2$.

$$\begin{aligned}
F^2 &= x_2(1-x_3)(1-r_D^2), \\
F_1^2 &= 1 - (1 - (1-x_3)(1-r_D^2))(1-x_1-x_2), \\
F_2^2 &= (1-x_3)(1-r_D^2)(x_1-x_2). \quad (13)
\end{aligned}$$

The expressions of $S_B(t)$, $S_T(t)$, $S_D(t)$ and S_t can be found in ref.[10, 14, 16, 17]. The wave functions of initial and final states can be found in ref.[4, 5, 8, 18–23].

With the functions obtained in the above, the amplitudes of these pure annihilation type decay channels can be given by

$$\mathcal{A}(B^0 \rightarrow D_s^- K_2^{*+}) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [a_2 \mathcal{A}_{af} + C_2 \mathcal{M}_{anf}], \quad (14)$$

$$\mathcal{A}(B_s^0 \rightarrow \bar{D}^0 a_2^0) = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} V_{cb}^* V_{us} [a_2 \mathcal{A}_{af} + C_2 \mathcal{M}_{anf}], \quad (15)$$

$$\mathcal{A}(B_s^0 \rightarrow D^- a_2^+) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us} [a_2 \mathcal{A}_{af} + C_2 \mathcal{M}_{anf}]. \quad (16)$$

III. NUMERICAL RESULTS AND DISCUSSIONS

For numerical analysis, we use the following input parameters:

$$\begin{aligned} f_{B/B_s} &= 0.21/0.23 \text{ GeV}, \quad f_{D/D_s} = 0.205/0.241 \text{ GeV}, \quad f_{K_2^*}^{(T)} = 118(77) \text{ MeV}, \\ f_{a_2}^{(T)} &= 102(117) \text{ MeV}, \quad M_{D/D_s} = 1.869/1.968 \text{ GeV}, \quad M_{B/B_s} = 5.279/5.366 \text{ GeV}, \\ |V_{cb}| &= 0.0415 \pm 0.0011, \quad |V_{ud}| = 0.9742 \pm 0.0002, \quad |V_{us}| = 0.2257 \pm 0.0012, \\ \Lambda_{QCD}^{f=4} &= 0.25 \text{ GeV}. \end{aligned} \quad (17)$$

After numerical calculation, the branching ratios of these decays are:

$$\begin{aligned} Br(B^0 \rightarrow D_s^- K_2^{*+}) &= (60.6_{-16.5}^{+17.3+4.3+3.2}) \times 10^{-6}, \\ Br(B_s \rightarrow \bar{D}^0 a_2^0) &= (1.1_{-0.4}^{+0.4+0.1+0.1}) \times 10^{-6}, \\ Br(B_s \rightarrow D^- a_2^+) &= (2.3_{-0.8}^{+0.8+0.2+0.1}) \times 10^{-6}. \end{aligned} \quad (18)$$

The branching ratio obtained from the analytic formulas may be sensitive to many parameters especially those in the meson wave function. For the theoretical uncertainties in our calculations, we estimated three kinds of them: The first errors in our calculations are caused by the hadronic parameters, such as the decay constants and the shape parameters in wave functions of charmed meson and the $B_{(s)}$ meson, and the decay constants of tensor mesons. The second errors are estimated from the unknown next-to-leading order QCD corrections with respect to α_s and nonperturbative power corrections with respect to scales in Sudakov exponents, characterized by the choice of the $\Lambda_{QCD} = (0.25 \pm 0.05) \text{ GeV}$ and the variations of the factorization scales defined

in eq.6 and eq.10. The third error is from the uncertainties of the CKM matrix elements. It is easy to see that the most important theoretical uncertainty is caused by the non-perturbative hadronic parameters, which are universal and can be improved by experiments.

These pure annihilation type decays considered in this work are dominant by W exchange diagram. All these decays do not have contributions from the penguin operators. Since the direct CP asymmetry is caused by the interference between the contributions of tree operators and that of penguin operators, it does not appear in these modes. Although the annihilation type diagrams are power suppressed in PQCD approach, the branching ratio of these considered Cabibbo-Kobayashi-Maskawa-favored decays are sizable and large enough to be measured in experiment. Through the study of these pure annihilation type decay modes, we can understand the annihilation mechanism in B physics well.

It is easy to find that there are large theoretical uncertainties in any of the individual decay mode calculations. However, we can reduce the uncertainties by ratios of decay channels. For example, simple relations among these decay channels are derived from eq.(14-16)

$$\begin{aligned}\frac{Br(B^0 \rightarrow D_s^- K_2^{*+})}{Br(B_s \rightarrow \bar{D}^0 a_2^0)} &\sim \frac{2f_{D_s}^2 V_{ud}^2}{f_D^2 V_{us}^2} \sim 60 \sim \frac{60.6}{1.1}, \\ \frac{Br(B_s \rightarrow \bar{D}^0 a_2^0)}{Br(B_s \rightarrow D^- a_2^+)} &\sim \frac{1}{2} \sim \frac{1.1}{2.3}.\end{aligned}\tag{19}$$

It is obvious that any significant deviation from the above relations will be a signal of new physics.

IV. SUMMARY

We calculate the branching ratios of three pure annihilation type decays in the perturbative QCD approach. The predicted branching ratios are $Br(B^0 \rightarrow D_s^- K_2^{*+}) \sim 6 \times 10^{-5}$, $Br(B_s \rightarrow \bar{D}^0 a_2^0) \sim 1 \times 10^{-6}$ and $Br(B_s \rightarrow D^- a_2^+) \sim 2 \times 10^{-6}$. They are sizable and large enough to be measured in forthcoming experiment. The study about the pure annihilation type decays can help us understand the annihilation mechanism in B physics. There are no direct CP asymmetries, because these decays have no contributions from penguin operators in the standard model.

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